

Quantum Oscillator HW 2B: Testing the order of different MATLAB ode solvers ode45, ode23, ode113 and ode15s

Computational Goal: For the second order ODE $\frac{d^2\phi_n}{dx^2} - [Kx^2 - \epsilon_n]\phi_n = 0$ with $K = 1$, $\epsilon_n = 1$ on the finite domain $x \in [-2, 2]$ and using the shooting method, find the order of convergence for different inbuilt MATLAB ode solvers

1 How do I set up the grid?

We use the shooting method in this assignment to solve the differential equation for the first eigen value only, on a smaller domain. We use different tolerance values for the ode solver as specified in the problem statement, instead of using the default tolerance as in Homework 1 (Important: This tolerance is the one set for ode function, not the tolerance within which the shooting method should match the boundary condition at $x = L$. Please do not get confused between the two!). This can be done using `odeset` to change the "RelTol" and "AbsTol" parameters, or within the `ode` function call itself. Also, in Homework 1 we set up the grid points in the domain $[-L, L]$ using a fixed grid spacing Δx and $n = \frac{2L}{\Delta x} + 1$ points. In this assignment, we just specify the domain endpoints or $xspan = [-2, 2]$ and the ode solver defines a grid that satisfies the tolerance specified. This grid may have fixed or variable grid spacing, remember that smaller tolerance values require finer grids.

2 How do I obtain the average grid spacing used by the solver?

The algorithm for the inbuilt ode solvers in MATLAB set up an initial grid, and keeps refining this grid till it meets the tolerance requirements. You have access to the grid the ode solver used by specifying the number of outputs the `ode` function returns, use help in MATLAB for details. The mean grid spacing can then be obtained using the `mean` and `diff` commands in MATLAB.

3 How do I compute the order of the ode solver?

Once you have the average step-size used by each of the 4 ode solvers for all the tolerance values, plot the logarithm of tolerance values on the y-axis versus the logarithm of the average step size on the x-axis. The slope of this line is approximately the local order of convergence of the ode solver.

Here is a summary of the steps:

1. Set $L = 2$, $xdomain = [-L, L]$, $\epsilon_n = 1$, and the initial values at $x = -L$ as defined, i.e. $\phi_n(-L) = 1$ and the corresponding value for $\frac{d\phi_n}{dx}$ at $x = -L$.
2. Define the tolerance values (TOL = 1e-4 ... 1e-10)
3. For each tolerance value solve using the shooting method just as in Assignment 1. When the shooting algorithm converges to the right boundary value at $x = L$, find the average grid size used by the ode solver
4. Find the slope of the log-log plot of tolerance v/s average step size. Hint: Look up the MATLAB function `polyfit`

Repeat for each ode solver.