

Model Thinking

Scott E Page

Why Model?

To design laws, rules,
and institutions.

Auctions

Objective

Get as much \$ as possible

Three Types of Auctions

Ascending Price

Second Price

Sealed Bid

Ascending Bid

Individuals call out bids until no one will bid a higher price.

Rational:

Psychological:

Rule Following:

Rational: Only bid up to your value

Psychological:

Rule Following:

Outcome: highest value
bidder gets it at the
second highest
value

100
70

Second Price Auction

Each person submits a bid.

Highest bidder gets it at the second highest bid.

Bidder 1: \$90

Bidder 2: \$60

Bidder 3: \$70

Pay \$70

Rational:

Psychological:

Rule Following:

Value = \$80, Bid = \$80

Highest Other Bid	Your Bid	Net
60	80	20
<u>75</u>	<u>80</u>	5
<u>85</u>	<u>80</u>	0

Value = \$80, Bid = \$90

Highest Other Bid	Your Bid	Net
60	90	20
75	90	5
85	90	-5

Value = \$80, Bid = \$70

Highest		
<u>Other Bid</u>	<u>Your Bid</u>	<u>Net</u>
60	70	20
75	70	0
<u>85</u>	70	0

Rational: Bid your true value

Psychological:

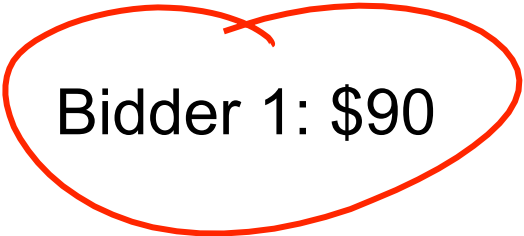
Rule Following:

Outcome: highest value
bidder gets it at the
second highest
value

Sealed Bid Auction

Each person submits a bid.

Highest bidder gets it at the highest bid.



Bidder 1: \$90

Bidder 2: \$60

Bidder 3: \$70

Rational:

Psychological:

Rule Following:

Higher you bid, the
more likely you
are to win.

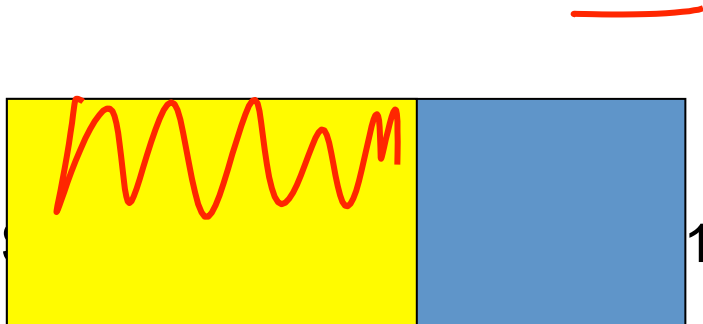
Two Bidder Model

Value of other bidder uniform in $[\$0, \$1]$



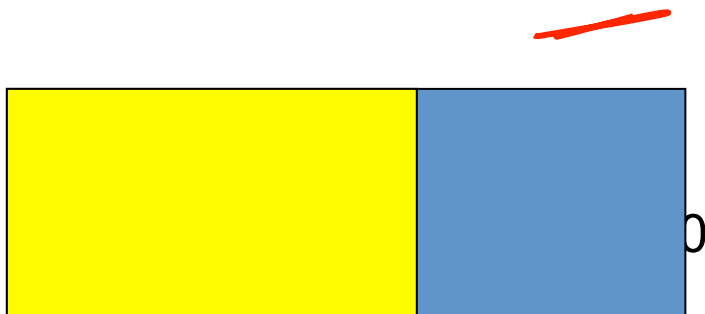
Scenario 1: Other bidder bids true value

$$\text{Probability}(\text{Bid} < .60) = 0.6$$



Scenario 1: Other bidder bids true value

$$\text{Probability}(\text{Bid} < B) = B$$



V = Value

B = Bid

$$\begin{array}{r} 90 \\ 30 \\ \hline 60 \end{array}$$

$(V - B) = \text{surplus}$

B = Probability of Winning

Winnings = $B(V - B)$

$$\text{Expected winnings} = B(V-B) = BV - B^2$$

Set Derivative = 0 to get a maximum

$$V - 2B = 0$$

Optimal Bid. $B = V/2$

$$V - 2B = 0$$

Rational:

Psychological:

Rule Following:

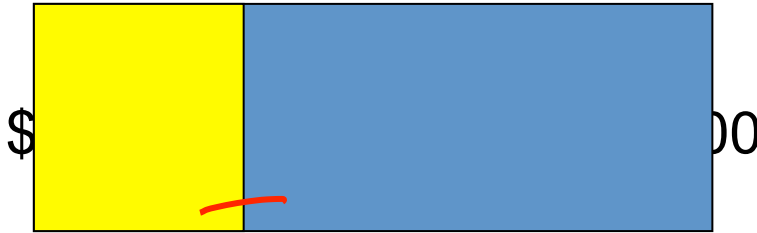
Rational: If I bid $V/2$, shouldn't other?

Psychological:

Rule Following:

Scenario 1: Other bidder bids half value

$$\text{Probability}(\text{Bid} < B) = 2B$$



$$v \leq (.25) + 2$$
$$= \frac{1}{2}$$

$V = \text{Value}$

$B = \text{Bid}$

$(V - B) = \text{surplus}$

$2B = \text{Probability of Winning}$

~~Winnings = $B(V - B)$~~

2

$$\text{Expected winnings} = 2B(V-B) = 2BV - 2B^2$$

Set Derivative = 0 to get a maximum

$$V - 2B = 0$$

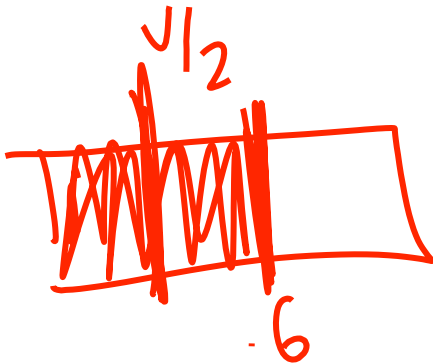
$$\text{Optimal Bid: } B = V/2$$

Outcome: highest value
bidder gets it half
her value

Sealed Bid: Highest value bidder wins at half her value

Ascending Bid: Highest value bidder wins at second highest value

Second Price: Highest value bidder wins at second highest value



Revenue Equivalence Theorem

With **rational bidders** a wide class of auction mechanisms including

- sealed bid
- second price
- ascending bid

produce identical expected outcomes.

Roger Myerson "Optimal Auction Design" *Mathematics of Operations Research* 1981

Rational:

Psychological:

Rule Following:

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