

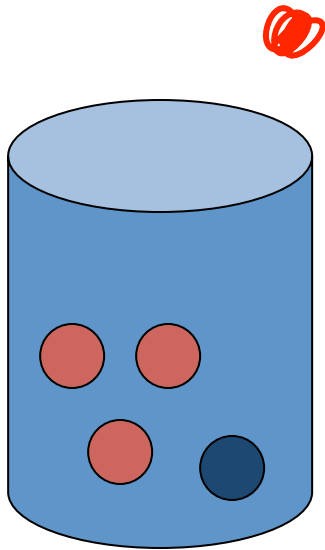
Model Thinking

Scott E Page

Urn Models

Basic Urn Model

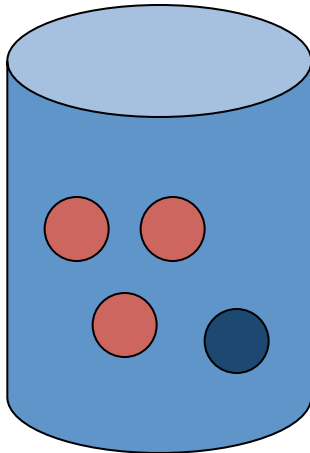
Urn contains balls of various colors. The outcome equals the color of the ball selected.



Bernoulli

$U = \{B \text{ blue }, R \text{ red}\}$
Select ball and return

$P(\text{red}) = R/(B+R)$
Outcomes **independent**

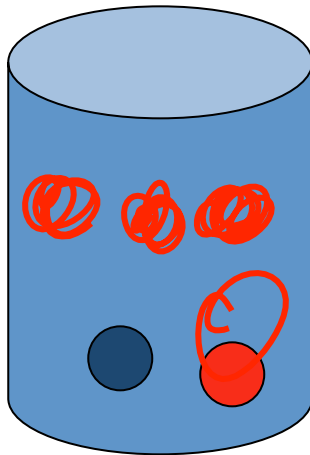


Polya

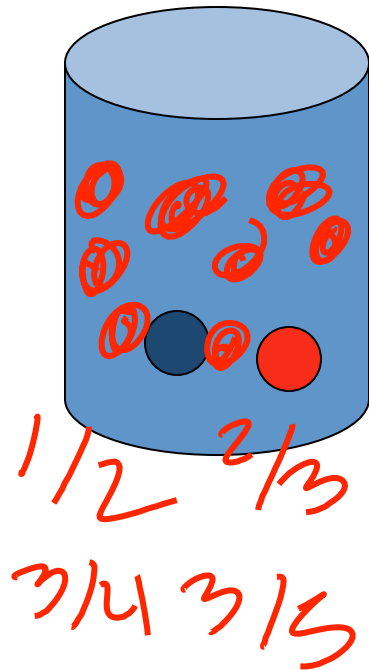
$U = \{1 \text{ Blue}, 1 \text{ Red}\}$

Select and return

Add a new ball that is the
same color as the ball selected



Polya in Action



Result 1: Any probability of red balls is an equilibrium and equally likely.

We could get 4%, 23%, or 99% red balls.

Result 2: Any
history of B blue
and R red balls
is equally likely

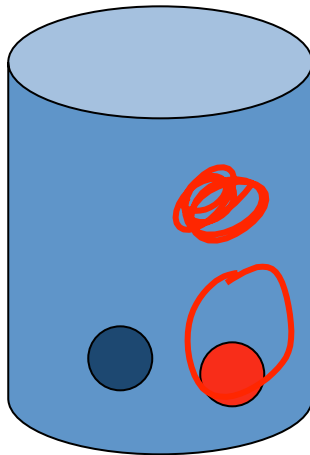
RRRB
BRRR

Balancing

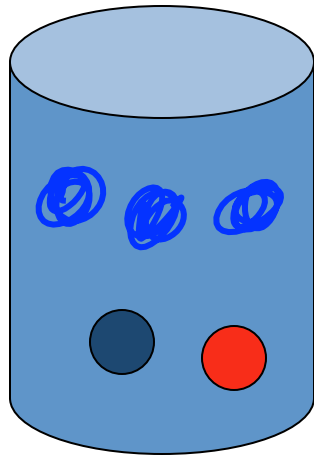
$U = \{1 \text{ Blue}, 1 \text{ Red}\}$

Select and return

Add a new ball that is the
opposite color as the ball selected



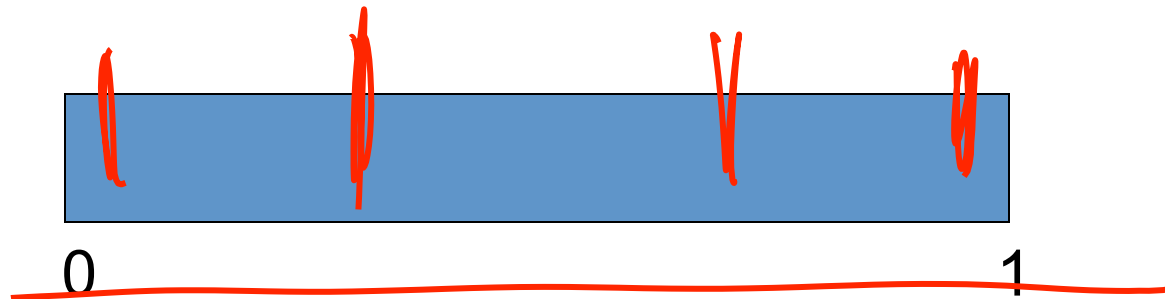
Balancing in Action



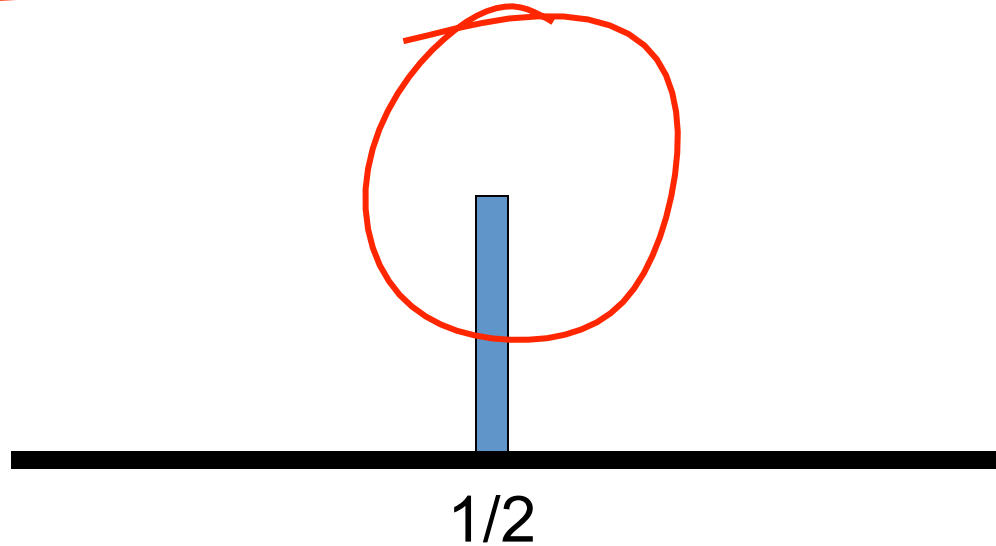
Result: The balancing process converges to equal percentages of the two colors of balls.

Polya vs Balancing

Polya:



Balancing



Period Outcomes:

color of ball in a
given period

Equilibrium:

percentage of red
balls in long run.

Path Dependent

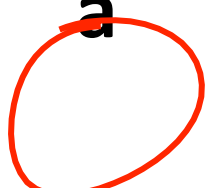
Outcomes: color of ball
in a given period depends
on the path

Path Dependent

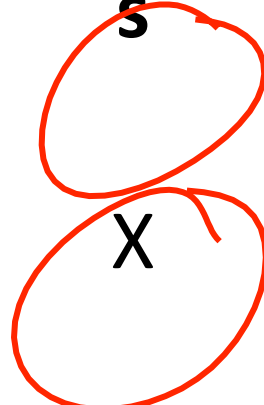
Equilibrium: percentage
of red balls in long run
depends on the path.

	Path	Path
	Depend	Depende
	ent	nt
	Equilibri	Outcome

a
X
1/2



s
X
X



Polya

**Balanci
ng**

History can matter at
each moment but not
have any impact in
the long run.

Examples: Manifest
Destiny, Railroads

Path Dependent:

Outcome probabilities
depend upon the
sequence of past
outcomes.



Phat Dependent:

Outcome probabilities
depend upon past
outcomes but not their
order.



PATH
PHAT

Polya is PHAT

Outcome probabilities do not depend upon the order in which outcomes occurred. They only depend on the set of outcomes that occurred.

RRB
BRR

Thirty rounds

$$\text{Paths} = 2 \times 2 \times 2 \times 2 \times 2 \dots \times 2$$

$> 1 \text{ billion}$

$$\text{Sets} = \{0, 1, 2, \dots, 30\}$$

$= 31$

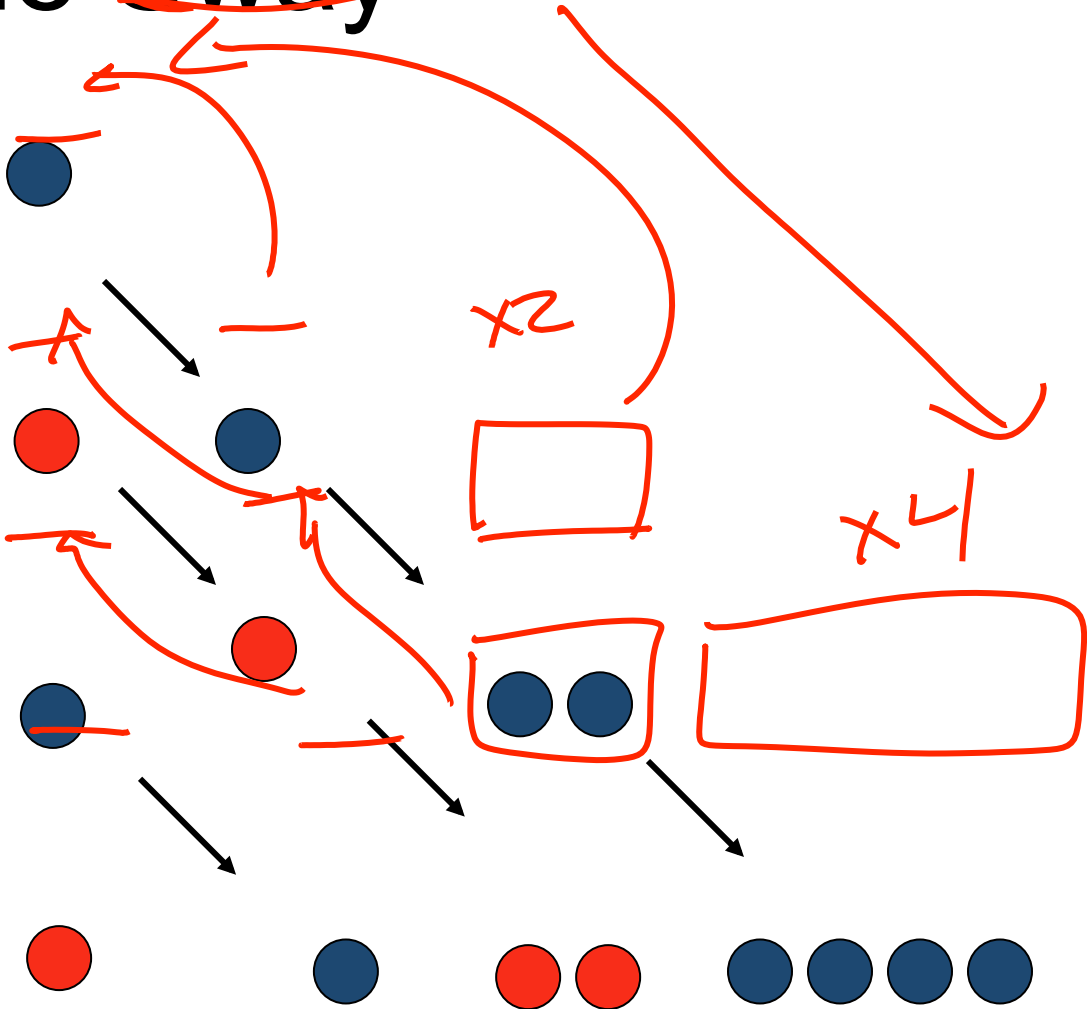
The Sway

$U = \{1 \text{ Blue}, 1 \text{ Red}\}$

Select and return

In period t , add a ball
of same color as
selected ball and add
 $2^{t-s} - 2^{t-s-1}$ of color
chosen in each period
 $s < t$

Sway



If the past takes on more weight over time then you can get full path dependence.

Model Thinking

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