

Model Thinking

Scott E Page

Simple Growth Model

Workers

Coconuts

Picking Machines

Machines wear out

L_t = Workers at time t

M_t = Machines at time t

O_t = Output of Coconuts at
time t

E_t = Number consumed at
time t

I_t = Number invested at
time t

s = savings rate

d = Depreciation rate

Assumption 1: Output is increasing and concave in labor and machines

$$\underline{O_t = \sqrt{L_t} \sqrt{M_t}}$$

Assumption 2: Output is consumed or invested

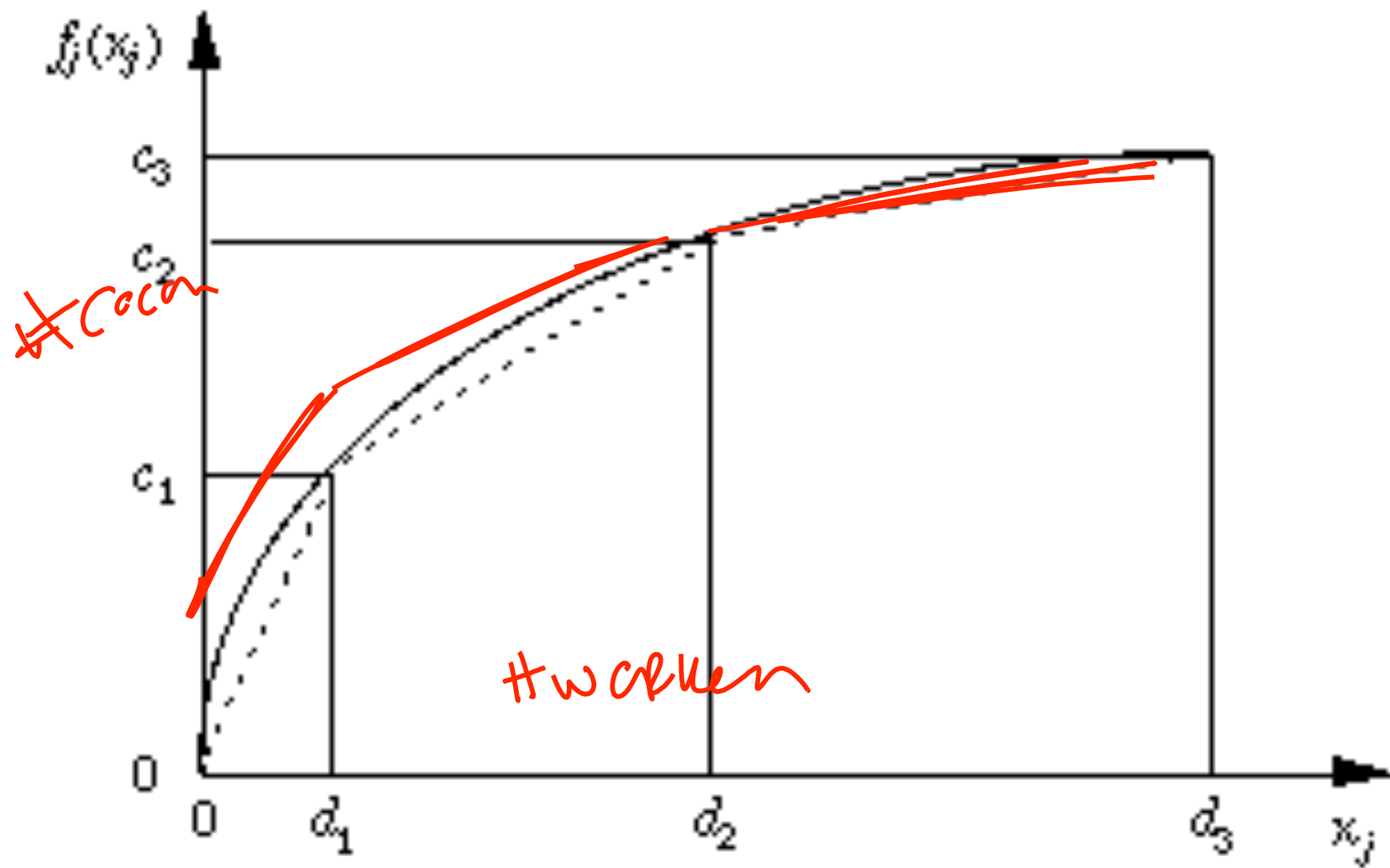
$$O_t = E_t + I_t \quad I_t = sO_t$$

Assumption 3: Machines can be built but depreciate

$$M_{t+1} = M_t + I_t - dM_t$$

Concave means that the first machine is worth more than the second, the second more than the third, and the third more than the fourth.

Economists call this **diminishing returns to scale**.



Simplification: $L_t = 100$

$$O_t = 10\sqrt{M_t}$$

$$\sqrt{L_t} \sqrt{M_t}$$

$$10\sqrt{M_t}$$

Example: $d = 0.25$, $s = 0.3$

Year 1: 4 Machines

Output = 10 $\times$ 4 = 20

Investment = 7.5

Depreciation = 9

$$20(0.3) = 6$$
$$\underline{4}(0.25) = \underline{1}$$

Year 2 Machines =

Example: $d = 0.25$, $s = 0.3$

Year 2: ~~9~~ machines

Output = 30 $\sqrt{9} = 9$

Investment = ~~2~~ $\frac{9}{4} = 2.25$

Depreciation = $9 \times 0.25 = 2.25$

Year 3 Machines =

$$30(0.3) = 9$$
$$9(0.25) = 2.25$$

Example: $d = 0.25$, $s = 0.3$

Year 3: 16 machines

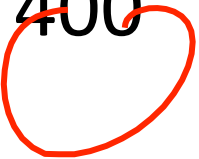
Output = $100 \sqrt{16} = 40$

Coconut GDP

Year 1: 200

Year2: 300

Year 3: 400



$$d = 0.25, s = 0.3$$

400 Machines?

$$\text{Output} = 10 \sqrt{400} = 200$$

$$(200)(.3) = 60$$

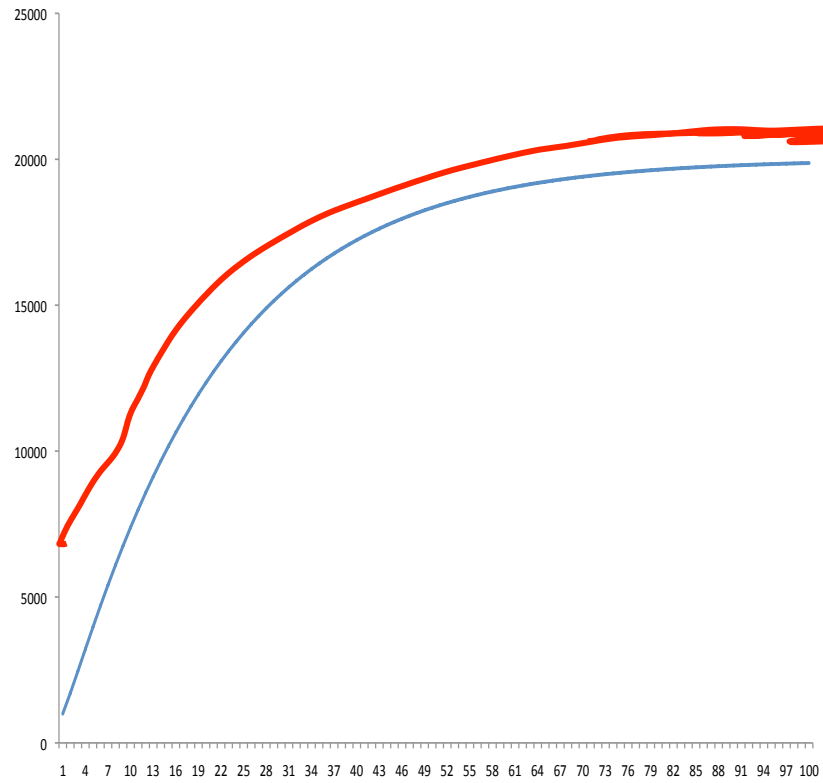
$$(400)\left(\frac{1}{4}\right) = 100$$

$$\text{Investment} = \frac{160}{-40}$$

$$\text{Depreciation} = 360$$

Machines =

Output



Equilibrium

Long Run Equilibrium:

Investment = Depreciation

Output = 10vM

Investment =

Depreciation = $3 \times 105m = 35m$
 $m/4$

$$35m = m/4$$

$$125m = m$$

$$12 = 5m$$

$$m = 144$$

$$\text{Output} = 10vM$$

$$\text{Investment} = 0.3 \times 10vM = 3vM$$

$$\text{Depreciation} = 0.25M$$

$$\text{Investment} = \text{Depreciation}$$

$$0.25M = 3vM$$

$$M = 12vM$$

$$M = 144$$

$$d = 0.25, s = 0.3$$

144 Machines

$$\text{Output} = 10 \sqrt{144} = 120$$

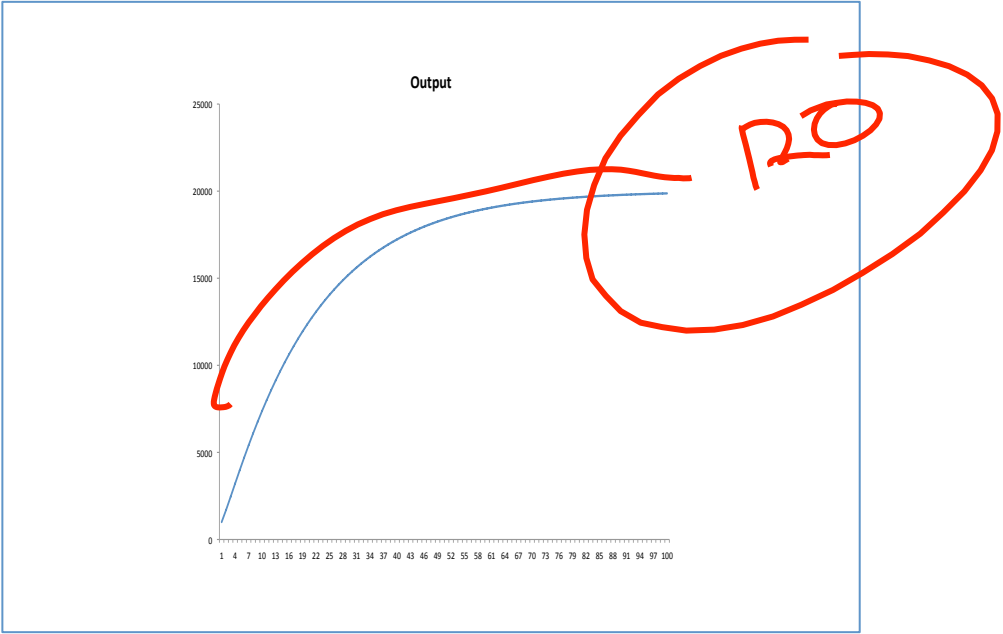
$$\text{Investment} = 36$$

$$\text{Depreciation} = 36$$

$$\text{Machines} = 144$$

$$(0.3)(120) = 36$$

$$\frac{1}{4}(144) = 36$$

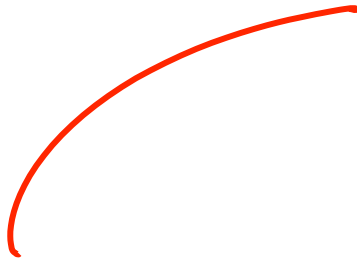


The Irony of the Growth Model



Depreciation: linear

Output: concave



Innovation

Model Thinking

Scott E Page