

Practice Problems 5: Solutions

$$1. \quad (a) \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 5 & 2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

A has 2 pivots.

$$(b) \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 5 & 2 & 8 \\ -1 & 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$x_1 + x_2 = 2$$

$$4x_2 + 2x_3 = 6$$

$$0x_3 = 2 \Rightarrow \text{No solution}$$

$$(c) \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 5 & 2 & 8 \\ -1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + x_2 = 2$$

$$4x_2 + 2x_3 = 6$$

$$0x_3 = 0 \Rightarrow \text{Let } x_3 = 1, \text{ then } x = (1, 1, 1)$$

$$(d) \quad \text{Let } x_3 = t, \text{ then } x = \left(\frac{1+t}{2}, \frac{3-t}{2}, t\right)$$

(e) No, A does not have an inverse; it does not have 3 pivots.

2. (a) Each matrix is $n \times n$

$$(b) \quad \begin{array}{l} AB = I \\ CAB = CI \\ IB = C \\ B = C \end{array} \quad \text{Or} \quad \begin{array}{l} CA = I \\ CAB = IB \\ CI = B \\ C = B \end{array}$$

(c) Yes, A is invertible, and $A^{-1} = B = C$

$$3. \quad (I - A)^2 = (I - A)(I - A) = I - 2A + A^2 = I - 2A + A = I - A$$

$$(I - A)^5 = (I - A)^2(I - A)^2(I - A) = (I - A)^2(I - A) = (I - A)^2 = (I - A)$$

$$4. \quad (a) \quad -3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \\ -5 \end{bmatrix}$$

(b) No, the coefficient matrix only has 2 pivots.