

Practice Problems 2: Solutions

1. (a) $\int x^3 \log(x) dx$

$$\begin{aligned} u &= \log(x) & dv &= x^3 dx \\ du &= \frac{1}{x} dx & v &= \frac{1}{4} x^4 \end{aligned}$$

$$\frac{1}{4} x^4 \log(x) - \frac{1}{4} \int x^3 dx = \frac{1}{4} x^4 \log(x) - \frac{1}{16} x^4 + c = \frac{1}{16} x^4 (4 \log(x) - 1) + c$$

(b) $\int x^2 e^x dx$ $x^2 e^x - 2 \int x e^x dx$

$$\begin{array}{ccc} u_1 = x^2 & dv_1 = e^x dx & \\ du_1 = 2x dx & v_1 = e^x & \Rightarrow \end{array} \quad \begin{array}{ccc} u_2 = x & dv_2 = e^x dx & \\ du_2 = dx & v_2 = e^x & \end{array}$$

$$x^2 e^x - 2 [x e^x - \int e^x dx] = x^2 e^x - 2x e^x + 2e^x + c = (x^2 - 2x + 2) e^x + c$$

(c) $\int [\log(x)]^2 dx$

$$\begin{aligned} u &= \log(x) & du &= \frac{1}{x} dx \\ e^u &= x & e^u du &= dx \end{aligned}$$

$$\int u^2 e^u du = (u^2 - 2u + 2) e^u + c = (\log^2(x) - 2 \log(x) + 2) x + c$$

2. (a) $\int_4^7 x^5 \log(x) dx$

$$\begin{aligned} u &= \log(x) & dv &= x^5 dx \\ du &= \frac{1}{x} dx & v &= \frac{1}{6} x^6 \end{aligned}$$

$$\frac{1}{6} x^6 \log(x) \Big|_4^7 - \frac{1}{6} \int_4^7 x^5 dx + c = \frac{1}{6} x^6 \left(\log(x) - \frac{1}{6} \right) \Big|_4^7 = 34055.10$$

(b) $\int_0^\infty \frac{1}{(1+x)^2} dx = \frac{-1}{x+1} \Big|_0^\infty = 0$

3. $g(x) = \frac{1}{\sqrt{2\pi}} \int_0^{b(x)} e^{-\frac{y^2}{2}} dy$ where $b(x) = \frac{1}{\sigma\sqrt{T}} \left[\log\left(\frac{x}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T \right]$

$$g'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{b(x)^2}{2}} b'(x) \quad \text{where } b'(x) = \frac{1}{\sigma x \sqrt{T}}$$