

Practice Problems 1: Solutions

$$1. \quad V = \frac{A}{r} \left[1 - \frac{1}{(1+r)^n} \right] \Rightarrow 437 = \frac{150}{r} \left[1 - \frac{1}{(1+r)^3} \right] \quad r = 0.014802$$

$$2. \quad (a) \quad \lim_{x \searrow 0} (2x)^{(2x)} = 1$$

$$(b) \quad \lim_{x \rightarrow \infty} \frac{1}{\sqrt{4x^2 - 2x - 10} + 2x} = 0$$

$$3. \quad (a) \quad f(x) = \frac{1}{5-x} = (5-x)^{-1}$$

$$f'(x) = (-1)(5-x)^{-2}(-1) = \frac{1}{(5-x)^2} \quad \text{where } x \neq 5$$

$$(b) \quad f(x) = \sum_{k=1}^{10} k e^{-a_k x^2}$$

$$f'(x) = \sum_{k=1}^{10} k (-2a_k x) e^{-a_k x^2} = -2 \sum_{k=1}^{10} k a_k x e^{-a_k x^2}$$

$$(c) \quad f(x) = \frac{\log\left(\frac{x}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$f'(x) = \left(\frac{1}{\sigma\sqrt{T-t}}\right) \left(\frac{K}{x}\right) \left(\frac{1}{K}\right) = \frac{1}{\sigma x\sqrt{T-t}}$$

$$(d) \quad f(x) = \frac{\log\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{x\sqrt{T-t}}$$

$$f'(x) = \frac{(x\sqrt{T-t})(x(T-t)) - \left(\log\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)\right)(\sqrt{T-t})}{x^2(T-t)}$$

$$= \frac{-\log\left(\frac{S}{K}\right) - \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{x^2\sqrt{T-t}}$$

$$4. \quad (a) \quad \lim_{x \searrow 0} \frac{\sin(3x^2)}{2x^2} = \lim_{x \searrow 0} \frac{6x \cos(3x^2)}{4x} = \lim_{x \searrow 0} \frac{6 \cos(3x^2) - 36x^2 \sin(3x^2)}{4} = \frac{3}{2}$$

$$(b) \quad \lim_{x \rightarrow \infty} \frac{\log(x^4)}{x} = \lim_{x \rightarrow \infty} \frac{4x^3/x^4}{1} = \lim_{x \rightarrow \infty} \frac{4}{x} = 0$$