



Introduction to Logic

Induction

Induction

Induction is reasoning from the specific to the general.

If various instances of a schema are true and there are no counterexamples, we are tempted to conclude a universally quantified version of the schema.

$$p(a) \Rightarrow q(a)$$

$$p(b) \Rightarrow q(b) \quad \rightarrow \quad \forall x. p(x) \Rightarrow q(x)$$

$$p(c) \Rightarrow q(c)$$

Incomplete Induction

Definition

$$f(1) = 1$$

$$f(x + 1) = f(x) + 2 * x + 1$$

Data

1	=	1	=	1 ²
1 + 3	=	4	=	2 ²
1 + 3 + 5	=	9	=	3 ²
1 + 3 + 5 + 7	=	16	=	4 ²
1 + 3 + 5 + 7 + 9	=	25	=	5 ²

Conjecture

$$f(x) = x^2$$

In this case, the answer is correct. Lucky Guess.

Not So Lucky Guess

Data:

$$2^{2^1} + 1 = 2^{2+1} = 5$$

$$2^{2^2} + 1 = 2^{4+1} = 17$$

$$2^{2^3} + 1 = 2^{8+1} = 257$$

$$2^{2^4} + 1 = 2^{16+1} = 65537$$

“Theorem” by Fermat (1601-1665):

$$\text{prime}(2^{2^x} + 1)$$

Fact discovered (mercifully) after his death:

$$2^{2^5} + 1 = 4,294,967,297 = 641 * 6,700,417$$

Oops.

Outline

Domain Closure

finite domains

Linear Induction

induction on sequences

Tree Induction

induction on trees

Structural Induction

induction on arbitrary structures

