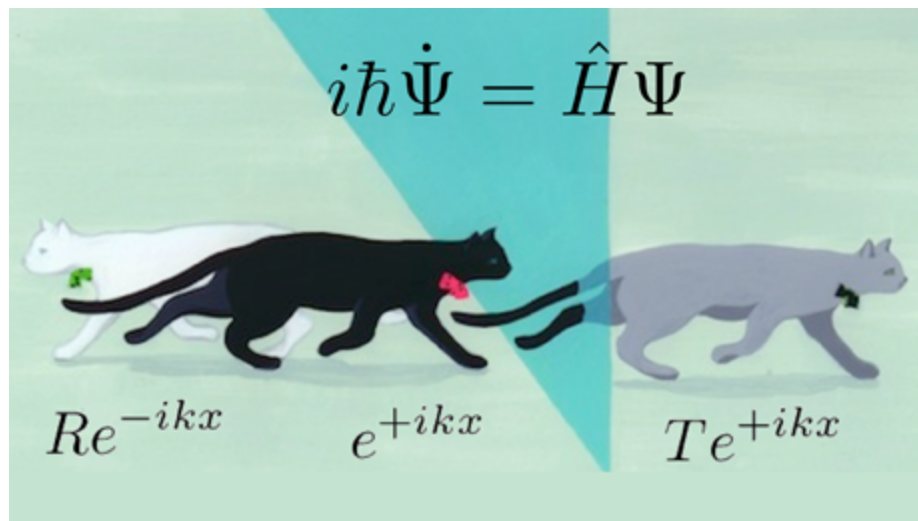


Angular Momentum

Part II. Basic commutation relations





Eugene Paul Wigner
1902-1995

1963 Nobel Prize in Physics

“for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles”.

Angular momentum

Now we derive the key commutator identities of the angular momentum operator, using the tools introduced in Part I.

You should work through these yourself. I will outline the steps.

First let's recall the definition of the angular momentum operator

$$\vec{L} = \vec{r} \times \vec{p} = \frac{\hbar}{i} \vec{r} \times \vec{\nabla} = \hat{\mathbf{e}}_i L_i = \frac{\hbar}{i} \epsilon_{uvw} \hat{\mathbf{e}}_u x_v p_w$$

Everything we do in this part requires just the use of the fundamental “uncertainty principle” commutator

$$[p_j, x_k] = p_j x_k - x_k p_j = \frac{\hbar}{i} \delta_{jk}$$

Try a simple application of this!

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Now let's calculate $[L_a, x_b] = L_a x_b - x_b L_a$

$$[L_a, x_b] = [\epsilon_{acd} x_c p_d, x_b] = \epsilon_{acd} \frac{\hbar}{i} x_c \delta_{bd} = i\hbar \epsilon_{abc} x_c$$

$$[L_a, x_b] = i\hbar \epsilon_{abc} x_c$$

$$\frac{1}{2} \left(\vec{L} \times \vec{x} + \vec{x} \times \vec{L} \right) = i\hbar \vec{x}$$

QUANTUM!

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$$[x_a p_j, x_k] = \frac{\hbar}{i} \delta_{jk} x_a$$

Now I recommend that you pause and work out the following identities:

$$[L_a, p_b] = i\hbar \epsilon_{abc} p_c$$

$$[L_a, L_b] = i\hbar \epsilon_{abc} L_c$$

$$\vec{L} \times \vec{L} = i\hbar \vec{L} \quad \text{QUANTUM!}$$

$$[L_i, L^2] = 0 \quad \text{where} \quad L^2 = L_i L_i$$